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The Empowerment of Problem-Based Learning Models to Improve Students' Quantitative Reasoning

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Abstract
Quantitative reasoning has been highlighted as essential for middle-school student's learning, particularly for themes that require students to make sense of relationships between quantities, according to a growing body of evidence. As a result, the current study adds to the body of literature that explores the growth of students' quantitative reasoning through teaching models. This study used measuring tools such as a quantitative reasoning test and an observation sheet. The randomized pre-test–post-test control group design had been used in this study. The study included 95 second-year middle school students from Pangkep, Makassar, South Sulawesi, who were split into two groups: experimental and control. The N-gain index, which had high, medium, and low categories, was used to calculate the improvement of students' quantitative reasoning exam results. The finding of the data shows that 86.0 % of students in the experimental class increase their quantitative reasoning exam scores in the high category, while only 46.6% of students in the control class improve their quantitative reasoning exam scores. Students' quantitative reasoning improves substantially more when they utilize the problem-based learning models to learn about the linear program than when they use direct learning. As a result, students' quantitative reasoning can be improved by using problem-based learning models.

Keywords: Problem-Based Learning, Quantitative Reasoning, Linear Program

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INTRODUCTION

The reasoning is a fundamental aspect of Mathematics (Sidenvall et al., 2015). To learn both procedurally and mathematically, reasoning students must practice how to solve both routine and non-routine tasks (Saleh et al., 2017; Sidenvall et al., 2015). Furthermore, by Ackerman and Thompson (2017) "Reasoning is the functioning of a person's schemes and operations in on-going interaction". The reasoning is one aspect of the goals of middle-school Mathematics. NCTM establishes benchmarks in the Mathematics learning process, which must include: problem-solving, reasoning and proof, communication, connection, and representation (Hasbi et al., 2019; Midgett & Eddins, 2001). This statement is in line with the objectives of learning Mathematics in Indonesia, one of which is that students have the ability to use reasoning on patterns and traits, perform mathematical manipulation in generating generations, compile evidence, or explain mathematical ideas and statements. The explanation states that the purpose of teaching Mathematics in middle school is to develop the ability to use reasoning. So that

Mathematics plays an important role in training students to reason in concluding to solve problems, both problems in Mathematics and problems in daily life-day. In both NCTM and Mathematics learning objectives, the reasoning is one of the abilities that must be achieved, especially quantitative reasoning.

Quantitative reasoning is known as quantity and its relationship. There is a problem with the quantity that everyone faces everyday. Quantity is the quality of something that is formed from the measurement process. Length, area, velocity, and volume are some attributes that can be measured in quantity (Ellis et al., 2019). One quantity can be related to other quantities, for example, speed is a quantity related to distance and time. When students engage in quantitative reasoning, students work with this quantity and quantity relationship (Ellis et al., 2019; Weber et al., 2014). For example, a student can compare amounts additively, by comparing how tall a person is with another, or by multiplying, by determining how many times larger one object is to another (Weber et al., 2014). Information that contains information in the form of magnitudes and their relation to other quantities is called quantitative information. From the point of view of educators and teachers, the main goal of Mathematics around the world is to link students' thinking with everyday problems that can be solved mathematically (Karim, 2007; Liljedahl et al., 2016; Santos-Trigo, 2020). This goal can be achieved by improving students' attitudes toward Mathematics. This increase according to Karim (2007) will occur if students can use quantitative reasoning. Reasoning consists of inductive, deductive, and quantitative (Carroll, 1992; Stanton, 1995). However, in research that is the focus of researchers, namely quantitative reasoning.

One of the most important mental processes for Math students is quantitative reasoning (Ellis et al., 2019; Moore, 2014; Weber et al., 2014). Quantitative reasoning by Dwyer et al. (2003), must be developed in Mathematics learning in order to students be able to assess quantitative data and determine the skills and methods that can be applied to specific issues in order to arrive at a solution or conclusion. Discussion of quantitative reasoning as a skill that all students must learn is one of the purposes of Mathematics education (NCTM, 2000). Quantitative reasoning has a variety of definitions in the literature. Quantitative reasoning, according to Dwyer et al. (2003); Weber et al. (2014), is a type of reasoning that stresses forming conclusions based on numerical data or information. Furthermore, quantitative reasoning is defined as the ability to represent quantitative data and to act on that data in order to reach previously unknown conclusions about the numbers represented or their connections (Moore, 2014; Nunes et al., 2015; Weber et al., 2014). Quantitative reasoning can be measured using four sorts of questions: quantitative comparison, multiple-choice (multiple-choice-select one), multiple-choice (multiple-choice select one or more), and enter answers in the box (numeric entry).

In any scenario, quantitative reasoning is defined as a network of quantitative numbers and relationships. When students reason quantitatively, they build quantities in the context of a situation. As a result, the process of measurement converts the attributes of a measurable object or phenomenon into a quantity (Ackerman & Thompson, 2017; Jack & Thompson, 2017). An individual or a student must be aware of the object, its measurable attributes, the suitable units of the measured attributes, and assign a numerical value to the measured qualities or features of the object that may be measured in the quantity construction process. Quantitative reasoning, in a nutshell, is thinking with quantities and their relationships (Kelly et al., 2015; Muzaini et al., 2019; Nunes et al., 2015).

A few research findings are related to this research. Like the findings of the study by Kabael & Akin (2018), quantitative reasoning has a favorable impact on students since it allows them to grasp and to construct excellent arguments based on quantitative facts, as well as to convey these ideas properly in various representations. Aside from that, quantitative reasoning has a significant role to play in the study of Mathematics in schools. Recent research results emphasize that Mathematics teachers have an important role in developing students' quantitative reasoning (Kabael & Akin, 2018). Thus, Mathematics teachers can use well-structured questions, such as asking the right questions, directing students to carefully consider quantitative relationships in problem situations, and thinking quantitatively are critical to improving students' quantitative reasoning in a learning environment (Ellis et al., 2019; Weber et al., 2014).

Based on the above description, a learning model that can integrate a mathematical problem connected to students' daily life is required to fulfill these objectives, so students are motivated and engaged in solving problems presented to them. The problem-based learning model is one learning model that can be used to help students practice problem-solving skills. It is feasible for pupils to develop problem-solving and quantity reasoning skills by focusing on the existence of an issue that they face connected to quantity problems. The problem-based learning model is excellent for students because it starts with contextual problems and asks well-structured questions, such as asking the proper questions, guiding students to carefully analyze quantitative linkages in problem situations, and thinking numerically.

By using unstructured problems that are extremely relevant to the subject and adopting a student-centered approach, problem-based learning (PBL) is one of the most common learning tools for helping students develop engagement and higher thinking (Abdullah et al., 2010; Pedersen & Liu, 2002; Sari et al., 2020). PBL models, according to Hmelo-Silver (2004), are models which share major pedagogical ideas that are divided into three categories: learning approach, social approach, and content approach. Furthermore, the PBL learning model is also characterized as "the process of learning to deal with problems involving identification, analysis, and solutions". Educators can employ made-up challenges to help students master specific abilities or real-life problems to provide them with meaningful learning opportunities. Therefore, these problems serve as the beginning and endpoints of the learning process (Hung, 2011; Mann et al., 2021). The selection of PBL models is also guided by the mathematical learning objectives, which include creating new methods of thinking or reasoning, developing creative activities, improving problem-solving skills, and conveying ideas. Problem-based learning (PBL) is a student-centered approach that has been examined as a delivery method by a number of higher education institutions throughout the world (Choon-Eng Gwee, 2008; English & Kitsantas, 2013). The problem-based learning model moves the emphasis of educational programs from teaching to learning, transforms students from passive consumers of knowledge into independent, active, and problem-solving learners (Akınoğlu & Tandoğan, 2007). Moreover, PBL learning encourages students to ask questions and to express ideas, to locate important knowledge from hidden sources in order to find diverse (alternative) approaches to solve problems and to determine the most effective solution.

Mathematical material and mathematical reasoning are two things that cannot be separated, namely Mathematics material is understood through reasoning, and reasoning is understood and trained through learning Mathematics material (Faradillah, 2018; Hasanah et al., 2019). Thus, reasoning skills play an important role in understanding and in solving mathematical problems. However, the facts on the ground are different from what they should be. Some previous research results show that Indonesia's quantitative reasoning ability is still relatively low. Hafiza et al. (2020), in his research, found that the students' quantitative reasoning ability is classified as a low category on trigonometry material. Students still have difficulties in answering questions in the form of quantitative reasoning abilities. Furthermore, Syarifuddin et al. (2019) discovered that 86,27% of students struggle with quantitative reasoning in the covariation problem process. While the results of the PISA study in 2012 showed that only 50.25% of Indonesian students were able to solve simple quantity problems, most Indonesian students were not able to solve more complex quantity problems (Firman, 2016).

Several factors influence these results, particularly the learning model, which is not appropriately utilized by teachers in order to explore students' reasoning abilities. As a result, a teaching model that can address the issue of low student quantitative reasoning is required. The Problem-Based Learning (PBL) model is one of the learning models that can increase problem-solving skills, particularly in quantitative reasoning difficulties. van der Vleuten & Schuwirth (2019) agree that the PBL model may be employed and is relevant to improving students' thinking, and Hung (2006); Kim & Kee (2013) support that the PBL model has the ability to improve an individual's or student's reasoning, which has an impact on learning outcomes.

The findings in the field include various even semester students in grade 8th in several Pangkaje'ne schools that are not the schools where the study originated. The author asks generalization questions on contextual problems to acquire a first impression. There are many assessment results that differ depending on the written results of pupils. The majority of students give up and are unable to solve the problem because they are unable to apply mathematical principles to the current circumstance. This is due to students' failure to connect the quantities in the problem at hand. This demonstrates that students' quantitative reasoning is deficient when they are unable to relate amounts, manipulate, and employ quantities in a consistent manner. Furthermore, some students presented nonsensical justifications for the relationships that occurred, while others grasped the concepts that were acceptable for the scenario but didn't know what to do to achieve the ultimate result. This is produced by a direct comparison approach error, and the last one is that students have trouble recognizing and creating genuine general statements. This demonstrates the importance of improving quantitative reasoning in order to improve students' generalization abilities, which have an impact on learning outcomes.

Some of the above-mentioned literature demonstrates the breadth of research into issues connected to students' mathematical reasoning in problem-based learning. Hence, the goal of this study is to see how a problem-based learning model might help students enhance their quantitative reasoning abilities while learning mathematics using PBL models. It concentrates on "linear program" material in particular.

METHODS

This study was a quasi-experimental research with randomized pre-test–post-test control group study designed to see how problem-based learning models enhance their student's quantitative reasoning abilities while learning Mathematics using PBL models. Students in Pangkep's 8th-grade middle-school school served as participants. The sample comprised 95 students from two grades, namely, grade 8.1 and grade 8.2. Grade 8.1 was selected as an experimental class taught using a problem-based learning model, with 28 female students and 22 male students, using random sampling techniques. Grade 8.2, on the other hand, is a control class that is taught utilizing direct learning models with an implementation based on the 2013 curriculum (direct learning). There are 25 female students and 20 male students in the control class.

A quantitative reasoning ability test sheet (QRATS) and an observation sheet were employed in the data collection instrument devised for this study. The QRATS test instrument comprised 5 questions with linear programming subjects that encompassed components of quantitative reasoning and was based on study findings Ramful & Ho (2015); Weber et al. (2014). Understanding the quantitative information in the problem, connecting the quantities contained in the problem or situation, looking for the same relationship between two or more quantities, looking for procedures to detect relationships between quantities, detecting patterns in a given situation, repeating patterns that exist to acquire a general form, and applying to new situations were all indicators of quantitative reasoning aspects. Additionally, when the teacher employed the problem-based learning model, the observation sheet was used to determine student’s actions. During the mathematical learning process, this page tracks student’s activities such as group discussions, questioning, and correct answers. The QRATS test instruments and observation sheets were validated by experts consisting of three lecturers and one Mathematics instructor before being used to collect data for this study.

The validator obtained an average of 3.87 with valid criteria after analyzing the findings of the QRATS instrument validation. Additionally, students were evaluated on the QRATS description test items to determine the instruments’ level of validity and reliability. Outside of the research subject, the QRATS test items were tested. Furthermore, the validity and reliability coefficients for field trials were 0.80 and 0.74, respectively. This means the instruments produced have a valid level of validity. As a result, the created instrument was suited for collecting data from quantitative reasoning ability tests.

The data obtained were then analyzed using a descriptive analysis by using quantitative approach. Furthermore, to determine the increase in the results of pretest of QRATS and posttest of QRATS, the calculation results about the tests were then categorized and matched with interpretation based on formulas and criteria in table two. The N-Gain formula and categorization used in this study is presented as follows (Hasbi et al., 2019).

$$<g> \equiv \frac{\% < G >}{\% < G >_{Max}} = \frac{(\% < S_f > - \% < S_i >)}{(100 - \% < S_i >)}$$

Table 1. N-Gain Value Criteria

N-Gain	Criteria
(<g>) ≥ 0.70	“High-g”
0,30 ≤ (<g>) < 0.70	“Medium-g”
(<g>) ≤ 0.30	“Low-g”

RESULTS & DISCUSSION

Description of the Implementation of the Problem-Based Learning Model in Learning Mathematics on the Topic of Linear Program

The results of observations of the implementation of learning during 4 meetings where at each meeting the implementation tend to be the same as applying the problem-based learning model. All phases in the initial activity are carried out to the maximum, at the core activities, there are 5 phases that are less than the maximum, namely: (1) the teacher guides students to identify problems by introducing students to what problems students will solve in learning activities, (2) the teacher arranges and organizes students in a learning task in accordance with the problem to be solved by students. Students are grouped and given learning assignments to solve problems, (3) the teacher guides students to collect data in general by conducting an investigation related to the problem being solved, both individually and in groups. During the learning process, students put forward many ideas related to the problem to be solved, these ideas are then discussed together both with groups and with the teacher, (4) the teacher guides students to communicate the results of their thinking or the results of their discussions related to solving mathematical problems in linear program material. Then presented in the form of a group assignment report of the material and questions discussed, and (5) the teacher helps students make an induction or checks related answers obtained. Where at this stage the teacher functions to analyze and to evaluate whether the problem solving that has been done by students is correct or not. The teacher also clarifies if there are mistakes made by students. In the test phase, the teacher gives quizzes or tests of quantitative reasoning ability (TKPK) to students by emphasizing to students to work on problems independently. In the final phase, everything is done. The core activities phase are not maximized because of insufficient time. However, the teacher still performs all the stages well when teaching students in class using the problem-based learning model.

Description of Students' Quantitative Reasoning Test Results

The process of conducting a study begins by giving of QRATS tests to both classes to see the level of students' initial quantitative reasoning ability. After applying problem-based learning, a test is given to see the improvement of students' quantitative reasoning ability. The control classes and the experimental classes are given the different learning model treatments.

Table 2 shows results QRATS to reveal if designed instruction creates a significant difference in students' quantitative reasoning in terms of pre-test and post-test scores. The following details are presented in the results of tests of quantitative reasoning abilities of students in both classes used as research samples.

Table 2. Results of QRATS Scores

Test	Class	Number of Subject	Highest Score	Lowest Score
Pre-Test	Experiment	50	70	45
	Control	45	65	40
Post-Test	Experiment	50	100	75
	Control	45	80	50

Table 2 above, it is found that the students' quantitative reasoning using Problem Based Learning (PBL) shows better results by looking at the average obtained by students in the experimental classes and the control classes. From this description, it can be stated that the students' quantitative reasoning using Problem-Based Learning (PBL) in experiments class increases and is better than students who get treatment with conventional learning or direct instruction in the control classes. Furthermore, the data obtained related to students' quantitative reasoning ability test scores is analyzed using normalized gain (N-gain). From the analysis results obtained a picture related to increasing the ability of quantitative reasoning students both from the experimental class and the control class described in the following table 3.

Table 3. N-Gain Results of QRATS

Class	Criteria	
	High (%)	Medium (%)
Experiment	86.0	14.0
Control	46.6	53.4

From the table above, it can be explained that the increase in students' quantitative reasoning abilities using the problem-based learning model looks significant where there is an increase in disability in the high category of 86.0% and an increase in the high category of 14.0% of medium category. Whereas for classes that apply the direct learning model, the increase in quantitative reasoning abilities in the high category is only 46.6% and 53.4% which is an increase in the medium category. This shows that the application of problem-based learning models in Mathematics learning can improve students' quantitative reasoning abilities.

Of the 50 students in the experimental class, 43 students meet the high criteria. Meanwhile, of the 45 students in the control class, 21 students meet the high criteria. A significance level of 0.05 is used to determine whether the proportion of students' quantitative reasoning scores is the same between the "experimental" and "control" classes. Based on the data, it shows that the proportion of students who meet the high criteria in the experimental class is 0.86 and the proportion of students who meet the high criteria in the control class is 0.46. Thus, the sampling distribution for the different test between the two proportions is to use the normal distribution. The null and alternative hypotheses of the two-proportion difference test are as follows:

H0: There is no difference between the proportion of students in "experimental class" and "control class"

H1: There is a difference between the proportion of students in "experimental class" and "control class"

Furthermore, the examination of the Z-table value of 1.985 (df = n-2) and the Z-count value of 5.127. As a result of Z-count > Z-table (5,127 > 1.985), H0 is rejected and H1 is accepted. This indicates that the proportion of students in the experimental and control classes differs, implying that the use of problem-based learning and direct learning models will result in different learning outcomes (quantitative reasoning).

Moreover, it is recognized that the teacher's learning activities match the good criteria at each level in PBL based on the outcomes of observations made during learning in the experimental class. The best time specified in the lesson plan for meeting the deadline for tolerance effectiveness and giving a good response to studying Mathematics in class is then used to mark student activities that fulfill the active criterion in learning. Students in early learning are given contextual problems to help them understand quantitative information about the problem, the relationship between the amounts in the situation, looking for the same relationship between two or more numbers, detecting interpersonal relationships, detecting patterns in certain situations, and repeating the pattern to get a general shape. As a result, students will expand their knowledge by integrating previous knowledge with current knowledge in order to draw inferences from the material presented. The teacher facilitates contextual problems in the learning process so that students are engaged because they may answer problems that are relevant to their daily lives and apply Mathematics skills. Students' abilities to interpret quantitative data and to discern the skills and methods that can be used to specific challenges to arrive at solutions or conclusions are also enhanced by the problems presented.

Students' responses meet positive criteria for learning so that students are motivated to build, to reason, and to develop scientific skills with better teacher's guidance. So that quantitative reasoning and learning experiences will be more meaningful and last in the long term. Students' responses include looking for the relationship between the amounts contained in the situation..

Observers observe instructor's and students' activities in learning activities that use problem-based learning (PBL). The following is an overview of the learning process obtained based on the observer's observations. Several meetings are held during the therapy stage or when learning is being implemented. Each meeting lasts 90 minutes, with an estimated learning time from the beginning to the end. In this study, the topic discussed in each meeting is linear programming material. Students are given contextual questions relevant to linear program material, in this example, the surface area and volume of the linear program, for each meeting, which they will discuss with their peers. During early learning activities, the teacher sets up the classroom for learning activities and communicates the material's objectives. Teachers are divided into groups depending on gender and students' aptitude levels, with the goal of having students undertake group discussions and solve problems that the teacher has presented.

Several students in each group ask the teacher questions concerning to the contextual concerns they explore throughout the material session. The teacher does not explicitly answer questions from students during the learning process; instead, the teacher just repeats the content and provides sufficient direction so that students can find the answers on their own. Furthermore, the teacher encourages students to investigate and to develop mathematical skills so that they are actively engaged in learning activities to better comprehend Mathematics in general. The teacher asks one representative from each group to deliver the results of the conversation and write them on the board when the group discussion is completed. The teacher analyzes whether the outcomes are consistent with the learning objectives after the group presents and presents the results of their discussion. In addition, the teacher will go over the results of each group's discussion in further detail. The teacher and students review and examine the problem-solving process at the conclusion of the lesson.

Before adopting Problem Based Learning, a number of researchers complete this study (PBL). The findings of this study, in general, can help students enhance their quantitative reasoning abilities. The findings show that when students use problem-based learning (PBL), their average quantitative reasoning improves and is superior to the control group. This occurs because the Problem Based Learning (PBL) learning method promotes and inspires students to seek solutions to issues they are given. Students can develop their ability to analyze quantitative information and to determine skills and procedures that can be applied to a particular problem to arrive at a solution or conclusion by using Problem Based Learning (PBL), which is associated with the concept of Mathematics itself, other scientific concepts, and other aspects of daily life (Dwyer et al., 2003; Muzaini et al., 2019).

Teachers can use learning theory to help children develop cognitively, socially, and spiritually. So far, four theories have guided the development of Problem Based Learning: Piaget's theory, Vygotsky's theory, Bruner's theory, and Ausubel's theory (Cassidy, 2004; Hasbi et al., 2019; McInerney, 2005). Learning and thinking evolve and create cognitive structures as a result of studying theory. Piaget proposed that a person's cognitive structure develops through a process of adaptation. Through two phases of assimilation and accommodation, adaptation is the process of adopting a scheme in response to the environment.

Because Problem Based Learning (PBL) focuses on how students think and reason rather than on student's accomplishment results, the learning process with PBL is closely tied to theory. This is based on Piaget's theory. Furthermore, students will be able to develop and to organize learning projects relevant to contextual difficulties (Ojose, 2008). Students gather relevant quantitative data and conduct experiments in order to explain and to solve problems. Vygotsky's theory is one of the most influential theories in the psychology of student's development. This approach emphasizes the societal nature of learning (Davydov, 1995; Eun, 2008; Gredler, 2012). Learning occurs when children labor or learn to complete tasks that have never been examined before but are still in the zone of proximal development. According to Vygotsky, higher mental functions emerge universally in conversation or cooperation amongst persons before being assimilated into the individual (Bendall et al., 2016; Cassidy, 2004; McInerney, 2005). This idea is based on Problem Based Learning (PBL), which is a contextual problem that is offered to students in order for them to understand and solve it in groups, as well as students planning and compiling relevant work outputs such as generating reports, models, and sharing assignments. With this model, each student will feel more secure and responsible for the information gathered during conversations, and students' quantitative reasoning will improve.

According to Bruner, Mathematics learning includes learning about mathematical concepts and structures contained in the material being studied, as well as determining the link between those concepts and structures (Brunner et al., 2009; McNeil & Uttal, 2009). The material will be fully comprehended thanks to these notions and structures. Furthermore, if the information being studied follows an organized pattern, students' knowledge is easier to recall and lasts longer. Furthermore, according to this theory, students' cognitive development is divided into three stages: the active stage, in which students learn to manipulate concrete items directly, and the iconic stage, in which students learn to manipulate symbolic concrete rather than concrete objects. The object stage is when students learn through manipulating symbols that aren't related to the item directly. As a result, Bruner's theory can be summarized as follows: problem-based learning, students' orientation to problems, students' organizations, conformity to individual and group research guidelines, thirdly developing and presenting work results, and suitability in analyzing and evaluating problem-solving processes. Some research findings that are pertinent to the findings of this study include the influence and rise in student's reasoning when problem-based learning is used (Gaze, 2018; Mkhatshwa, 2020; Stocker et al., 2021; Sumartini, 2015; Tallman & Frank, 2020). However, the results of this study also contrast with the results of several previous studies (Firman, 2016; Hafiza et al., 2020; Syarifuddin et al., 2019).

Implications for Future Research

So the sample size in this study is rather limited, future research should focus on increasing the length of time and frequency of students' participation. The purpose of this study is to look into students' broad quantitative reasoning abilities. More research with larger samples and longitudinal studies are needed to ensure that problem-based learning models are actually successful in developing students' quantitative reasoning abilities. Further research is required to assess the significance of problem-based learning models in increasing students' quantitative reasoning and adding insight, particularly for teachers' human resources.

CONCLUSION

According to the findings of this study, 86 percent of 8th-grade middle-school students in Pangkep experience an increase in quantitative reasoning test scores in the high category in the experimental class, while the increase in quantitative reasoning test scores in the control class is in the medium category, with a percentage of 46.6 percents. This suggests that when it comes to the linear quantitative reasoning program, students who use problem-based learning models outperform those who utilize direct learning models by a large margin. These findings show that selecting the correct learning model can increase students' cognitive skills and ability to learn Mathematics. This study looks into whether using a problem-based learning model can help students enhance their quantitative reasoning abilities. This learning model allows students to answer Math issues involving quantity by applying mathematical principles learned in prior sessions to situations that they encounter in their daily lives. Furthermore, this model allows students to create mathematical concept knowledge based on the outcomes of discussions with other students and the teacher during the learning process. As a result of these interactions, students will endeavor to learn and to grasp all of the prerequisite knowledge and concepts employed in the problem-solving process, which will improve students' quantitative reasoning abilities.

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