



Prospective Mathematics Teachers' Beliefs and Representations

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Abstract

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Beliefs have been identified as an affect on the psychology of Mathematics education. Additionally, representations play a paramount role in Mathematics teaching and learning. However, few studies have examined prospective teachers' beliefs and their relationship to representations in problem-solving. The purpose of this study is to examine prospective Mathematics teachers' beliefs and representations in solving word problems reviewed from gender differences. This study involves 103 college students from the Mathematics education department at a public university in Merauke Regency. The data were gathered using a questionnaire, a test, and in-depth interviews. A questionnaire was utilized to elicit the belief of prospective teachers. A test and interview were conducted to evoke data about participants' representations when solving word problems. The male student was found to be capable of arriving at a solution employing multiple representations. Due to the inaccurate representation constructed, the female student was unable to find the correct solution. This work implies that lecturers should heed prospective teachers' beliefs and generate representations of the focal point of classroom learning practice.

Keywords: Belief, Gender, Prospective Teacher, Representations.

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INTRODUCTION

Representation is defined as an entity or configuration of characters, images, concrete objects that can symbolize or represent something else (Goldin & Shteingold, 2001; Mitchell et al., 2014; Stylianou, 2011). Representation can refer to processes and products, namely activities or processes, to understand mathematical concepts or the relationships between these concepts (NCTM, 2000). Representation has become a topic of study in the last few decades and proved that representation has a crucial role in understanding mathematical ideas and solving mathematical problems (Andhani, 2016; Jupri et al., 2020; Pagiling, 2019). Representations that both teachers and students construct can be material for discussion in class and help teachers monitor students' attention when understanding a particular concept. In this case, representation is a tool to facilitate discussion in class (Stylianou, 2010). In addition, the representations built and used by students when solving problems can be used by teachers as an assessment tool to gain insight into students' reasoning (Santia et al., 2019).

Most of the current evidence supports the use of representation to promote students' correct solutions in solving Math problems (Boonen et al., 2014; Selling, 2016; Yee & Bostic, 2014). Students who generate and employ multiple representations also foster inflexible thinking skills in problem-solving and help them to understand mathematical concepts (Brizuela et al., 2015; Hwang et al., 2007). In addition, students who construct representations of a concept or use representations when solving problems

naturally tend to reduce the level of abstraction of the given problem and adjust to the cognitive structure they already have (Panasuk & Beyranevand, 2010).

On the other hand, belief is one of the psychological domains that influence a person is viewing an event that is considered accurate. Ernest (1989) explains that teachers or prospective teachers who hold an instrumentalist view would see Mathematics as a tool consisting of an accumulation of formulas, rules, procedures, or skills that can be used everywhere. Platonist teachers will see Mathematics as a static but unified science of specific knowledge. Because of this, concepts in Mathematics are interrelated and logically related, and teachers who view problem-solving will see Mathematics as not a finished product but a process of human creation and invention in solving problems. Therefore, Mathematics is constantly evolving and open to revision. Teacher's belief play a paramount role in the planning and the implementation of classroom teaching, likewise with the belief of a teacher who can foster learning efforts to increase student's engagement in constructing an understanding of mathematical concepts.

Several researchers have scrutinized that teachers' beliefs about the nature of Mathematics affect the practice of teaching and learning Mathematics in the classroom (Beswick, 2005; Safrudiannur & Rott, 2019; Siswono et al., 2017; Viholainen et al., 2014). Conceptually the quality of classroom teaching is determined by the teacher's decisions, which are influenced by the interaction between knowledge and belief. Concerning mathematical problem solving, this interaction is studied by Siswono and colleagues (2017) on three lower secondary school Mathematics teachers who find that teacher beliefs have a strong relationship with teachers' knowledge of problem-solving. In particular, the instrumental-type teachers' beliefs in their research are consistent with their insufficient problem-solving knowledge, while the platonist-type teachers' beliefs and the problem-solving-type teachers' beliefs are respectively consistent with adequate problem-solving abilities.

Several studies have emphasized that pre-service teachers and in-service teachers do not fully understand the strategic role of representation for learning and teaching mathematics (Dreher et al., 2016; Pagiling & Munfarikhatin, 2020; Rosli et al., 2013). The teacher candidate's conception or understanding of representation will influence them to perceive the role of representation in learning Mathematics and problem-solving. On the other hand, prospective teachers' belief about nature Mathematics and Mathematics teaching and learning is associated with how they approach and solve mathematical problems (Safrudiannur et al., 2021). Beliefs about Mathematics can be divided into two categories: students' beliefs in viewing Mathematics as a science, such as how to think Mathematics, see non-routine mathematical problems, and perceive the usefulness of Mathematics and beliefs about how students feel first beliefs (Yeo, 2009).

Research on representations associated with belief has not been widely documented. Moreover, the current literature proves the lack of studies on representations based on gender. Since gender is one of the domains of psychology can contribute to Mathematics achievement, it is essential to explore the prospective teachers' beliefs and representations in terms of gender. Acknowledging that representations play an essential role in Mathematics, learning and practicing can enhance an essential first step for any targeted ability intervention to remove the gender gap and ensure gender equity. Therefore, the current study investigates the beliefs and prospective representations of Mathematics teachers in solving word problems reviewed from gender.

METHODS

This study was descriptive that used quantitative and qualitative approaches. The instruments used include the TBTP questionnaire adopted from (Safrudiannur & Rott, 2019), mathematical representation tests, interview guidelines, and recording devices. The TBTP questionnaire was given to all Mathematics education students studying at Musamus University to determine students' beliefs about Mathematics and beliefs about learning and teaching Mathematics. After the prospective Mathematics teachers (PMTs) accomplish the TBTP, the data is then analyzed to map students' beliefs. To explore the mathematical representation, PMTs are asked to do a mathematical representation test. This test is a word problem consisting of three numbers containing fractions, Geometry, and number patterns. The word problems were chosen since the information on the word problems was not written explicitly, so the PMTs needed representation to illustrate the information. This present study only discusses the work of PMTs in solving fraction problems and PMTs' beliefs about representation. The fraction word problem is given to the participants that would evoke discussion on how they represent mathematically can be displayed as follow.

The Fractions Word Problem

Mrs Anti went to the market and had spent half her money on rice. After that, she spent another two-thirds of the remaining money on fish. Determine the money that Mrs Anti brings to the market if the remaining money is IDR 20,0000.00

Two PMTs comprised of one male and one female who had the same cumulative grade point average and had platonist beliefs were interviewed to dig deeper into the representations constructed in solving word problems about the application of fractions. We recruited only two students who represent platonist beliefs since they had similar abilities. The interview used was a semi-structured interview that aimed to determine the mathematical representation constructed and used by prospective teachers in problem-solving. The interviews were recorded using smartphones to ensure no information was missed and the data obtained was guaranteed to be valid. The results of the interview recordings were transcribed in detail, which were then combined with field notes and the results of the researchers' observations while conducting interviews. Then triangulated the data to test the credibility of the data. We utilized the triangulation technique from test and interview techniques to ensure credibility. The data validity was checked on the same subject with a different technique, namely checking the consistency of the mathematical representation test data with interviews. Moreover, we performed a member check by providing each participant with their narrative (Creswell, 2015). All participants agreed that we accurately captured and described their representations.

Data analysis was carried out by data reduction, focusing on the mathematical representation of teacher candidates, presenting data representations in figures and narrative words, and drawing conclusions. In verifying the conclusions, investigator triangulation was carried out (Rothbauer, 2008). All authors were actively involved in data collection and verification of the findings to reach a consensus conclusion.

RESULTS & DISCUSSION

This section initially displays the 103 prospective Mathematics teachers' beliefs about the nature of Mathematics. Subsequently, we present a prospective male

Mathematics teacher and a prospective female Mathematics teacher's representations who have platonist beliefs in solving fraction word problems.

Results

a. PMTs' Belief

In the following, data on 103 PMTs' beliefs about the nature of Mathematics are presented.

Table 1. PMTs' Belief

Items	Belief	Mean	Standard Deviation
1	Instrumental	6,36	1,02
	Platonist	5,31	1,15
	Problem-solving	4,69	1,31
2	Instrumental	5,94	1,12
	Platonist	5,47	1,22
	Problem-solving	4,98	1,31
3	Instrumental	5,92	1,24
	Platonist	5,29	1,25
	Problem-solving	4,60	1,53
4	Instrumental	5,93	1,09
	Platonist	5,24	1,26
	Problem-solving	4,71	1,54

Table 1 shows that most participants in this study give rates associated with instrumentalist beliefs. Prospective teachers' instrumentalists view Mathematics as a set of tools made up of facts and rules to solve various problems. Moreover, the instrumentalist view is that Mathematics accumulates facts, rules, and skills to pursue some shallow end (Beswick & Chick, 2019; Philipp, 2007). Thus, Mathematics is a set of unrelated but utilitarian rules and facts. Meanwhile, PMT responds to the item associated with the problem-solving view. Prospective teachers' problem-solving views Mathematics as a space for human discovery that is continuously developing and where patterns emerge and are subsequently filtered to become knowledge. Moreover, the problem-solving teachers view Mathematics as a dynamic, continuously expanding field of human creation and invention, a cultural product. Mathematics is a process of inquiry and coming to know, not a finished product, for its results remain open to revision.

b. Representation of Prospective Male Mathematics Teacher (PMMT)

The representations constructed and used by a prospective male teacher in solving word problems are presented in Figure 1.

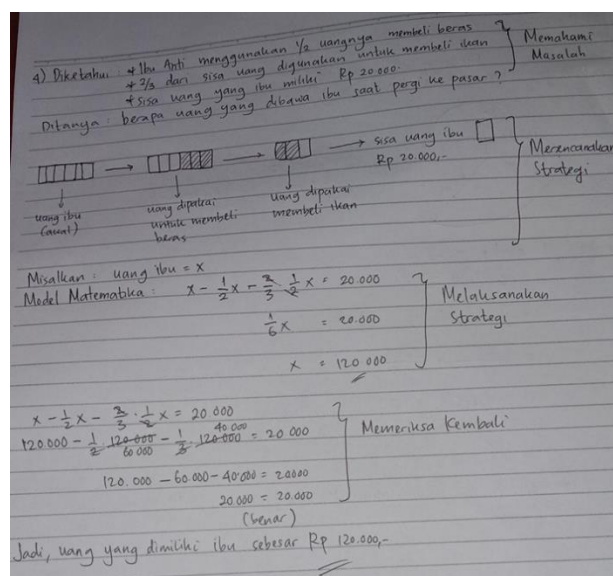


Figure 1. PMMT' Representations

The researcher's interview (R) transcript with the prospective male Mathematics teacher (PMMT) is presented in dialog 1.

Dialogue 1

- R : What do you know or what information is available in the problem?
 PMMT : From the problem, it is known that Mrs Anti uses half of her money to buy rice. Then $\frac{2}{3}$ of the money is used to buy fish so that the remaining money is IDR 20.000
- R : So, what is unknown from this problem?
 PMMT : The amount of money that Mrs Anti brought when she went to the market.
- R : From this information, how do you plan to solve the problem?
 PMMT : I will draw an image in the form of a rectangle to make it easier to illustrate the information in the problem.
- R : What is the image like?
 PMMT : I will draw three rectangles. The first image refers to the initial amount of money that Mrs Anti brought to the market. Then, the second image illustrates half of the money that Mrs Anti uses to buy rice. The third image refers to $\frac{2}{3}$ of the remaining money used to buy fish.
- R : Good. Why did you use a rectangle image?
 PMMT : To make it easier to model the amount of Mrs Anti's money.
- R : After you finish drawing, what are your next steps?
 PMMT : I generate a mathematical model.
- R : What is the mathematical model like?
 PMMT : The initial amount of Mrs. Anti's money is symbolized by x . Then, I subtract x with $\frac{1}{2}x$ and then subtract $\frac{2}{3}$ times $\frac{1}{2}x = 20.000$ ($x - \frac{1}{2}x - \frac{2}{3} \cdot \frac{1}{2}x = 20.000$)
- R : What do your next step?
 PMMT : I completed a mathematical model to get Mrs. Anti's initial money.
- R : Good. How much money does Mrs. Anti have?
 PMMT : 120.000,- IDR
- R : Are you sure? How do you check the truth?

- PMMT* : Yes, I am sure it is true. I substituted $x=120,000$ into the mathematical model.
- R* : I guess there is another way, right?
- PMMT* : Yes, sir. From the rectangle model, the $1/6$ model is sufficient to solve the problem since $1/6x=20,000$. Thus, we get $x=120.000$.

Figure 1 and dialogue 1 prove that the PMMT solves fractional problems using multiple representations (verbal, visual, and symbolic representations). He understands the information available in the problem by using a rectangle model. He constructs rectangles to illustrate information so that it becomes concrete. In particular, he subdivides a whole area of rectangles into six equal parts to portray Mrs Anti's money. Subsequently, he shades three parts out of six or $1/2$ parts of the whole which refers to the remaining money from Mrs Anti after buying rice. Next, he constructs a $2/3$ fraction model that depicts $2/3$ part of $1/2$ the initial amount of Mrs Anti's money, $1/3$ of the money used to buy fish. Next, PMMT formulates a problem-solving strategy by connecting the known and unknown information and writing it down in a mathematical model: $x-1/2x-1/3x=20,000$. In figure 1, PMMT executes the plan by subtracting x by $1/2x$ and $2/3 \times 1/2 x$. Finally, he gets $1/6x=20,000$ or $x=120,000$.

The PMMT arrives at the correct solution since it can construct an accurate visual representation. In addition, he can construct the correct mathematical models in solving fraction word problems. Moreover, he has another strategy to cope with the problems using a rectangle model without expressing mathematical notation. He constructs rectangle manipulatives to represent a fraction. He certains that rectangle which represents $1/6 x$ is equivalent to 20.000. From the explanations above mentioned, it can be concluded that the PMMT can depict information on the problem using various representations and has flexible thinking in solving word problems. These findings corroborate the previous studies (Nugroho et al., 2020; Wibawa et al., 2020), which scrutinize the flexibility in using representations to ensure students' success in solving word problems.

c. Representation of Prospective Female Mathematics Teacher (PFMT)

The representations constructed and used by a prospective female teacher in solving word problems are presented in Figure 2.

Ibu Anti pergi ke pasar dan telah membelanjakan separuh uangnya untuk membeli beras. Setelah itu, ia membelanjakan lagi dua pertiga dari sisa uang tersebut untuk membeli ikan. Tentukan uang yang dibawa Ibu Anti ke pasar jika uang yang tersisa adalah Rp 20.000,00.

Jawab:

Dik: Misal, uang Ibu Anti x : x
 Sisa Uang = 20.000

Dit: $x = \dots$?

Peny: * Sisa belanja : $1 - \frac{1}{2}$
 : $\frac{1}{2}$ bagian

* Sisa uang : $\frac{2}{3}$ bagian

* Sisa uang = 20.000

$\frac{1}{3} \times x = 20.000$

$x = 20.000 \times 3$

$x = 60.000$

Jadi, uang awal Ibu Anti adalah Rp 60.000,00.

Figure 2. PFMT' Representations

Dialogue 2

- R : Please read the problem. What information is provided in the problem?
- PFMT : From the question, it is known that Mrs Anti has used half of her money to buy rice, but we do not know the initial money. Then $\frac{2}{3}$ of the money is used to buy fish.
- R : $\frac{2}{3}$? Where did this money come from?
- PFMT : From the remaining money, Mrs Anti bought rice
- R : Then, what was asked?
- PFMT : The initial amount of money that Mrs Anti had.
- R : From this information, what is your strategy in solving problems?
- PFMT : First, I will suppose the money with variables like x , y , and z .
- R : What refers to x ?
- PFMT : x is the initial amount of Mrs Anty's money.
- R : What is the next step?
- PFMT : Write x as a unified whole. Then reduce by $\frac{1}{2}$, which is the money used to buy rice; you get $\frac{1}{2}$. Then, I multiply $\frac{1}{2}$ with $\frac{2}{3}$, and we get $\frac{1}{3}$.
- R : What $\frac{1}{3}$ does represent?
- PFMT : $\frac{1}{3}$ represents Mrs Anti's remaining money. We get $\frac{1}{3} x = 20.000$. Thus, $x = 60.000$.
- R : What did the x refer to?
- PFMT : x is The initial amount of Mrs. Anti's money.
- R : Do you have solved the problem?
- PFMT : Yes, sir.
- R : Are you sure about your solution?
- PFMT : Yes, I am sure.
- R : How do you check the correctness?
- PFMT : I substituted $x = 60.000$ into the equation that was created.

Figure 2 and dialogue 2 show that the PFMT solves the fraction problem using verbal and symbolic representations. She understands the information available in the problem using verbal sentences. Furthermore, she supposes x as the initial amount of Mrs Anti's money. The PFMT formulates a problem-solving strategy by connecting the known and the

unknown information. However, the symbolic representation building is not accurate. It can be seen in figure 2 that PFMT executes the plan by subtracting one by $\frac{1}{2}$ to get $\frac{1}{2}$ which refers to Mrs Anti's remaining money has after buying rice. Subsequently, she multiplies $\frac{1}{2}$ by $\frac{2}{3}$ to get $\frac{1}{3}$. She concludes that $\frac{1}{3}$ of the x (original money) is the rest of the spending money, so she gets $\frac{1}{3}x = 20,000$ or $x = 60,000$. She does not subtract x by $\frac{1}{2}x$ and $\frac{1}{3}x$ to show Mrs Anti's remaining money. PFMT's solution is not accurate since it is not able to make the correct mathematical model in solving fraction word problems. It can be seen that the PFMT does not have flexibility in employing representations. These results agree well with existing studies on flexibility representations in solving word problems (Aziz & Kurniasih, 2019; Fonger, 2019; Lee & Lee, 2019).

Discussion

The findings of this study are that prospective male Mathematics teachers write detailed solutions using multiple representations in solving fraction word problems. The PMMT uses visual, verbal, and symbolic representations flexibly. At the stage of understanding the problem, the PMMT understands the problem by describing the information available on the problem using a visual representation, recognizing general information by constructing a picture. In addition, at this stage, MPMT uses verbal representations; namely, PMMT understands problems using spoken and written words. He develops problem-solving strategies using verbal and visual representations in the planning stage. He expresses problem-solving plans or strategies by using verbal representations in the form of spoken words. In addition, PMMT expresses a problem-solving plan or strategy by using a symbolic representation by using a mathematical model.

Furthermore, PMMT executes the plan using a formal notation representation by revealing the manipulation of mathematical models at the plan's stage. The PMMT interprets the results of the solution verbally. While at the re-examination stage, PMMT checks the correctness of the solution by using a symbolic representation by substituting the value of x into a mathematical equation. These findings corroborate that somewhere in the process, the solver must formulate a (mental) mathematical problem from the problem situation, which allows him to employ and to choose the appropriate mathematical representations and calculations towards a mathematical solution of the problem (Boonen et al., 2013; Hoogland et al., 2018; Selling, 2016).

On the other hand, prospective female Mathematics teachers write solutions with incomplete understanding using verbal and symbolic representations to solve fraction word problems. In other words, since the female teacher-candidate does not have a visual image in her mind, she has difficulty solving the problems (Giardino & Wöpking, 2019; Nugroho et al., 2020). At the stage of understanding the problem, the PFMT apprehends the problem by describing the provided information on the problem using a verbal representation; namely, the FPMT grasps the problem using spoken and written words. FPMT develops problem-solving strategies using verbal and symbolic representations in the planning stage. FPMT expresses problem-solving plans or strategies by using verbal representations in the form of spoken words. In addition, FPMT expresses problem-solving plans or strategies by using symbolic representations of mathematical equations.

Furthermore, the FPMT executes the plan made using a formal notation representation at the implementation stage. She is completing a mathematical model to arrive at the solution. Furthermore, the FPMT interprets the results of the settlement verbally. At the looking-back stage, FPMT checks the correctness of the solution using symbolic representation by substituting the value of x into the equation.

We only interview two prospective teachers with the same belief. Different results might be obtained if a similar study involves prospective teachers with a different belief. However, we argue that the findings of this study provide theoretical implications of teacher candidates' beliefs about representation in learning that have not been comprehensive. Most prospective Mathematics teachers consider Mathematics a tool for solving problems (Beswick, 2012; Philipp, 2007; Safrudiannur & Rott, 2020). Prospective teachers' instrumentalists view Mathematics as a set of tools made up of facts and rules to solve various problems. The results obtained here may have implications for understanding the problem solver's belief influence students' transition between representational forms and convert them into different forms were more successful in problem-solving.

CONCLUSION

Most prospective Mathematics teachers have instrumental beliefs that view Mathematics as a toolset of facts, principles, rules, and skills. The prospective male Mathematics teacher succeed in solving word problems since he could employ various forms of representation. On the other hand, the prospective female Mathematics teacher does not arrive at the correct solution because of her dependence on one form of representation. This study suggests to the Mathematics teachers educators to prepare prospective teachers by habituation to the practice of pedagogical representation in the classroom.

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